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ATMOSPHERIC MODELS AND ASTRONOMICAL AND PARALLACTIC REFRACTION

by

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[redacted]

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ABSTRACT The problem of the refraction angle of rays passing through the atmosphere from an object in space to a ground observer is examined for cases where 1) the object is within the atmosphere (high-altitude target), 2) the object is beyond the atmosphere in near space (satellite, meteor), and 3) the object is the infinity (star). Theoretically this investigation is based on work performed by L. Oterma [Astr. Inst. Univer. d. Turku, Informo No. 20 Turku1960], A. A. Baldini [GIMRADA, Res. Note No. 8 Fort Belvoir, Va. 1963], andJ. Kabelac [Wiss. Z. Techn. Univ. Dresden, 14(1965) H. 3. Data for the construction of the atmospheric models are taken from COSPAR InternationalReference Atmosphere 1961]. A distinction is made in the various cases of the problem between astronomical refraction and R_∞ and parallactic R_H. It is found that astronomical refraction R_∞ is not dependent on the momentary state of the atmosphere, while the expression $R_{\Delta H}$ and ΔR_H, characterizing ray deviation when passing through a specific layer ΔH or to a specific height H, are very dependent on the existing state of the atmosphere, and therefore on time. The dependence is of the order of 1" for a zenithal distance of 45°. The influence is indirectly proportional to the height of the target above the place of observation and directly dependent on the air density. An analogous dependency obtains in the case of parallactic refraction, if the object is inside the atmosphere. Original Art has: 12 formulas, 13 tables, and 3 figures.

ATMOSPHERIC MODELS AND ASTRONOMICAL AND PARALLACTIC REFRACTION

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1. Introduction

The object of the present work is to determine the refraction angles of the rays passing through the atmosphere from a body to the observer. The body can be inside the atmosphere (high-altitude target), outside of the atmosphere near the earth (meteor, artificial satellite). or in infinity (a star). Therefore we will distinguish the astronomical refraction R_∞ (Fig. 1) and the parallactic refraction $\bar{R}_H$ (Fig. 2) for near targets outside and inside the atmosphere. Besides this, we will give our attention to the refraction $\Delta R_{\Delta H}$ of a light ray in its passage through an air layer which is not bound by the earth's surface and to refraction ΔR_H in passing through an air layer which is bound on one side by the earth (Fig. 1). In order to make it possible to interpret these formulas in a simple way on different atmosphere models (Part 4), the formulas are arranged so that the expressions which one can present exactly in analytical form can be distinguished from those expressions that are empirical. The calculation of these expressions is done by the method of numerical integration.

It is not the purpose of the work to investigate the refraction changes dependent on the atmospheric conditions in the immediate vicinity of the measurement station. The work proceeds theoretically

mainly from [1] and partially also from [2], and represents an extension of work [3]. As a basis for the construction of the atmospheric models we have used [4], particularly the Soviet data for the standard atmosphere contained therein. In the conclusions there are some data added regarding the paraliactic refraction.

2. Astronomical Refraction

2.1 Derivation of a Formula Suitable for the Calculation of the Astronomical Refraction through Numerical Integration

Under the assumption that the refraction of a light ray coming from a star occurs in accordance with the laws of refraction, and that the earth's atmosphere can be replaced by spherical, concentrated, homogeneous layers – independently of whether we later may perceive that this assumption is a disadvantage for the generality – we can set up the basic formula for astronomical refraction R_∞ (Fig. 1).

$$R_\infty = \int_{n=1}^{n_0} \sin z' : \{(n^2 r : n_0 r_0)\left[1 - (n_0 r_0 : nr)^2 \sin^2 z'\right]^{\frac{1}{2}}\}\, dn\,, \qquad (2.1)$$

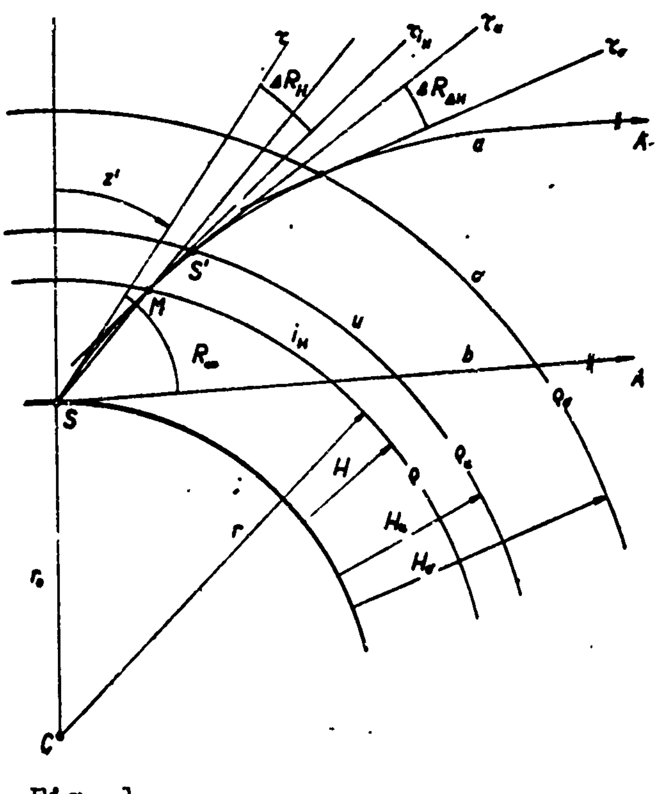

Fig. 1.

where r is the radius of the overall spherical surface, r_0 is the distance of the station S from the earth's center C, n and n_0 are the absolute refraction indices of the spherical layers with radii r and r_0, and z' is the zenith distance of star A which has no refraction. Therefore, R_∞ is the angle formed by the tangent τ to light ray a at point S with direction b to star A without atmospheric influences. Further, we introduce the foci n and r and the new variables v and s with the help of the expressions $n_0:n = 1 + v$ and $r_0:r = 1 - s$, where v and s are very small positive values.

In order to get closer to the atmospheric models, which are principally characterized by variation in density, we introduce, instead of the absolute refraction index n, air density ρ with the help of the Gladston-Dale law $1 = n - k\rho = n_0 - k\rho_0$, where k is a constant and ρ_0 the air density at distance r_0 from the earth's center C, i.e., at the location of the station S. Let us substitute these expressions in (2.1) and integrate within limits $\rho = 0$ to $\rho = \rho_0$. Then after conversion we get the resultant for astronomical refraction:

$$R_\infty = k(2n_0)^{-1}\,\mathrm{tg}\,z'[2\varrho_0 + 2\varrho_0\,\mathrm{lg}\,n_0 - {}^1S + \mathrm{tg}^2 z'(\varrho_0\,\mathrm{lg}\,n_0 - {}^1S + 1{,}5\,{}^2S) + \\ + \mathrm{tg}^4 z'(1{,}5\,{}^2S - 3\,{}^vS)]. \tag{2.2}$$

where

$$\begin{aligned} {}^1S &= 2\int_0^{\varrho_0} s\,\mathrm{d}\varrho = \sum_{i=0}^{n-1}(s_i + s_{i+1})\,\Delta\varrho_i, \\ {}^2S &= 2\int_0^{\varrho_0} s^2\,\mathrm{d}\varrho = \sum_{i=0}^{n-1}(s_i^2 + s_{i+1}^2)\,\Delta\varrho_i, \ldots, \\ {}^vS &= 2\int_0^{\varrho_0} sv\,\mathrm{d}\varrho = \sum_{i=0}^{n-1}(s_i v_i + s_{i+1}v_{i+1})\,\Delta\varrho_i, \end{aligned} \tag{2.3}$$

where $\Delta\rho_i = \rho_i - \rho_{i+1}$. The subscript i indicates the i-spherical surface.

2.2 The Numerical Values in the Formula for Astronomical Refraction R_∞

Before the numerical values of the necessary constants are inserted into equation (2.2) we will compare them with [1, 4-9]. The

value $n_0 = 1.00027687 \pm 2 \cdot 10^{-7}$ was introduced into the further calculations for normal temperature and pressure.

Also the values of the absolute freezing point T_{abs} and the coefficient of the air-volume expansion $\alpha = 1:T_{abs}$ were taken from sources as independent as possible. We have used the values $T_{abs} = 272.8°C$ and $\alpha = 0.0036656$ [1, 4, 6, 8].

The air density ρ_0 was taken from [4] and it amounts to $\rho_0 = 1.2250 \times 10^{-3}$ g/cm^3 for $t_0 = 15°C$ and for $p_0 = 760$ mm Hg. To these values there corresponds $k = 0.22602$ (the constants in the Gladston-Dale law), which is in good agreement with [10].

Analysis of the precision in the constants. For the required precision of 0.01" in the resulting astronomical refraction R_∞, and in fact up to zenith distance of 76°, it is necessary that n_0 be set at $6 \cdot 10^{-9}$ precisely, α at 10^{-10}, ρ_0 at $2 \cdot 10^{-5}$, and k at 10^{-5}. Except the expressions n_0 and α, the precision of the remaining constants is attainable, or it is at the limit of the requirements. Knowledge of n_0 to the 7th decimal place is somewhat problematical and uncertain. This could be one of the main reasons why, in the selection of the method of geodetic astronomy, preference is given to those procedures where measurement is done in small zenith distances or where the systematic refraction component is broken down, as is the case, for example, in the Talcott-method, in the method of equal elevations, in the Zinger method, and so on. The expressions ${}^1S$, ${}^2S$, and ${}^\nu S$ were calculated in accordance with the trapezoid rule and for the sake of checking in accordance with the Simpson rule. In the computation, Soviet data on the earth's standard atmosphere were used [4]. The closest gradation of the argument was selected so as to correspond to altitude differences of 100 m, for the lowest strata. With the increasing elevation the gradation of the argument increased to 20 km for elevations over 100 km, where the atmospheric influence on the value of the astronomical refraction is almost meaningless. The integration was carried through to the elevation of 200 km. For r_0, the value for the average earth diameter 6,368.8 km [1] was taken. In Table 2.1 some total values ${}^1S$, ${}^2S$, ${}^\nu S$ are given at elevations H.

We designate them as calculated from (2.3) as, ${}^1S_H$, ${}^2S_H$, and ${}^{\nu}S_H$. The summation values up to the elevation of 200 km are ${}^1S = 3.2592 \cdot 10^{-6}$, ${}^2S = 7.404 \cdot 10^{-9}$, and ${}^{\nu}S = 0.655 \cdot 10^{-9}$. After insertion in equation (2.2) and after adjusting, we have

$$R_\infty = 57{,}0326'' \operatorname{tg} z' - 0{,}067\,79'' \operatorname{tg}^3 z' + 0{,}000\,213'' \operatorname{tg}^5 z', \tag{2.4}$$

where for a zenith distance to 20° it is sufficient to consider the first term in (2.4), from 20° to 60°, the first two terms, and from 60° to 76° all three.

TABLE 2.1.

H km	ϱ g/cm³	s	${}^1S_H$ g/cm³	${}^2S_H$ g/cm³	ν	${}^{\nu}S_H$ g/cm³
0	$1{,}2250 \cdot 10^{-3}$	0	0	0	0	0
5	$7{,}3654 \cdot 10^{-4}$	$784 \cdot 10^{-6}$	$3\,584 \cdot 10^{-10}$	$0{,}9 \cdot 10^{-10}$	$1{,}104 \cdot 10^{-4}$	$0{,}3 \cdot 10^{-10}$
10	4,1357	1 567	10 995	9,8	1,834	1,4
15	1,9467	2 349	19 419	26,2	2,33	3,2
20	$8{,}8870 \cdot 10^{-5}$	$3\,130 \cdot 10^{-6}$	25 110	41,6	2,57	4,5
25	4,0621	$391 \cdot 10^{-5}$	28 601	52,4	2,68	5,5
30	1,7901	469	30 529	61,5	2,73	6,0
40	$4{,}0003 \cdot 10^{-6}$	624	31 996	69,4	2,76	6,4
50	$1{,}0754 \cdot 10^{-6}$	$779 \cdot 10^{-5}$	32 396	72,2	2,77	6,5
60	$3{,}3162 \cdot 10^{-7}$	$93 \cdot 10^{-4}$	32 522	73,3	2,77	6,5
70	$9{,}2747 \cdot 10^{-8}$	109	32 570	73,8	2,77	6,5
80	$2{,}0979 \cdot 10^{-8}$	124	32 587	74,0	2,77	6,6
90	$3{,}4733 \cdot 10^{-9}$	139	32 591	74,0	2,77	6,6
100	$5{,}3993 \cdot 10^{-10}$	155	32 592	74,0	2,77	6,6
155	$2{,}0521 \cdot 10^{-12}$	238	32 592	74,0	2,77	6.6
200	$4{,}4301 \cdot 10^{-13}$	$304 \cdot 10^{-4}$	$32\,592 \cdot 10^{-10}$	$74{,}0 \cdot 10^{-10}$	$2{,}77 \cdot 10^{-4}$	$6{,}6 \cdot 10^{-10}$

The density ρ will be determined mainly in the lower strata from the meteorological data on the state of the atmosphere, which is characterized by the instantaneous temperature t, and the pressure p, and in fact in accordance with the state equation of an ideal gas

$$\varrho = \varkappa p/T, \tag{2.5}$$

where κ is a constant and T the temperature on the Kelvin scale. The constant κ was determined from the Soviet data [4] for the given ρ, p, and T from equation (2.5); for the Hg pressure given $\kappa = 4.6447 \cdot 10^{-4}$.

2.3 Evaluation of Some Formulas for the Astronomical Refraction R_∞

In accordance with formula (2.4) the values of R_∞ were calculated for the different zenith distances and in Table 2.2 are compared with

like values calculated from other formulas. From Table 2.2 there is to be seen good agreement between the formula here derived (2.4), from the Bessel formula [6], and the most used formula[2] R, for the computed refraction of simplest form. The values of R_∞ from the Oterma formula [1] give lower values; with increasing zenith distance the difference becomes greater. In [11] there is a comparison of the separate development terms of (2.4) and the Oterma development [1] and it is seen that the development of (2.4) to a certain extent in the area z' from 0° to 76° converges more rapidly, especially for short zenith distances, which is caused by the introduction of the function tg instead of second. The difference values in [1] are clearly brought about by other numerical values of the basic constants. Altogether one can state that the introduction of real values of ρ_i of the atmosphere into the formula for R_∞ brings no new results. It apparently comes about in this way that eventual density anomalies in the lower strata of the atmosphere in the sums ${}^1S$, ${}^2S$, and ${}^\nu S$ do not show up in the same measure as if the observed body were in the atmosphere.

TABLE 2.2.

Literature z'	R	[6]	[1]	Eq. (2,4)
10°	10,06"	10,06"	10,01"	10,06"
20	20,75	20,77	20,67	20,76
30	32,91	32,94	32,78	32,91
40	47,83	47,85	47,62	47,82
50	67,89	67,90	67,57	67,85
60	98,40	98,61	98,03	98,43
70	155,20	155,45	154,69	155,32

3. Refraction of Light Ray in Passing Through Air Layer

3.1 Derivation of Formula Suitable for the Computation of Numerical Integration

By refraction $\Delta R_{\Delta H}$ of the light ray a on passing through the layer of air we understand the angle (Fig. 1) formed by tangents τ_o and τ_u to light ray a at the intersections of this ray with the

upper o and lower u boundaries. This refraction takes in only the course of the ray to the station S. The value of $\Delta R_{\Delta H}$ is determined through integration of equation (2.1) from ρ_o to ρ_u, which represent the density at the elevations H_o and H_u. After inserting the numerical values we get

$$\Delta R_{\Delta H} = 23\,303{,}2'' \operatorname{tg} z' \{2{,}001\,107(\varrho_u - \varrho_o) - {}^1S_{\Delta H} + \\ + \operatorname{tg}^2 z' [0{,}000\,553\,7(\varrho_u - \varrho_o) - {}^1S_{\Delta H} + 1{,}5^2 S_{\Delta H}] + \operatorname{tg}^4 z' [1{,}5^2 S_{\Delta H} - 3^\nu S_{\Delta H}]\} . \quad (3.1)$$

where

$$ {}^1S_{\Delta H} = \sum_{i=u}^{o} (s_i + s_{i+1})\,\Delta\varrho_i , \quad {}^2S_{\Delta H} = \sum_{i=u}^{o} (s_i^2 + s_{i+1}^2)\,\Delta\varrho_i , \\ {}^\nu S_{\Delta H} = \sum_{i=u}^{o} (\nu_i s_i + \nu_{i+1} s_{i+1})\,\Delta\varrho_i . \quad (3.2)$$

The value for the refraction $\Delta R_{\Delta H}$ becomes apparently predominant from the densities ρ_u and ρ_o at the elevations H_u and H_o, and not until then will it depend on ${}^1S_{\Delta H}$, ${}^2S_{\Delta H}$, and ${}^\nu S_{\Delta H}$.

Further, we determined the refraction for the case where the body M is within the atmosphere at height H (Fig. 1), and station S is on the surface of the earth. Let us designate it as ΔR_H. It is namely the angle which the tangent τ_{i_H} forms at the point a of intersection of the light ray and area i_H with the tangent τ to the light ray at the station S. It will thus be a matter of a stratum for which H_o = H and H_u = 0. The change in the integration limits is $\rho_u \to \rho_0$ and $\rho_o \to \rho$, where density ρ belongs to the height H.[3] After inserting the numerical values we get the final form for ΔR_H:

$$\Delta R_H = 23\,303{,}2'' \operatorname{tg} z' \{0{,}002\,4507 - 2{,}001\,107\varrho + 0{,}452\varrho^2 - {}^1S_H + \\ + \operatorname{tg}^2 z' [0{,}000\,000\,339 - 0{,}000\,5537\varrho + 0{,}226\varrho^2 - {}^1S_H + 1{,}5^2 S_H] + \\ + \operatorname{tg}^4 z' [1{,}5^2 S_H - 3^\nu S_H]\} , \quad (3.3)$$

where

$$ {}^1S_H = \sum_{i=0}^{iH} (s_i + s_{i+1})\,\Delta\varrho_i , \quad {}^2S_H = \sum_{i=0}^{iH} (s_i^2 + s_{i+1}^2)\,\Delta\varrho_i , \\ {}^\nu S_H = \sum_{i=0}^{iH} (s_i \nu_i + s_{i+1} \nu_{i+1})\,\Delta\varrho_i . \quad (3.4)$$

Summation values (3.2) and (3.4) are worked out through numerical integration or are taken from the tables in [11] – see also Table 2.1.

The same operations are valid for them as for the sums ^{1}S, ^{2}S, and $^{\nu}S$. In the case where the body M is at an elevation greater than 10 km it is not necessary to take into account the expressions with ρ^2 in equation (3.3). One can conclude from equation (3.3) that the refraction value ΔR_H will depend mainly on ρ^*, i.e., on the temperature t and on the pressure p at the place of the body M in accordance with equation (2.5), and only after this on the density of the air along the path of the light ray between station S and target M. In order to assure accuracy in $\Delta R_{\Delta H}$ and in ΔR_H to 0.01" up to the zenith distance of 76° it is necessary that the density values ρ_u, ρ_o, and ρ in formulas (3.1) and (3.3) be made accurate to $5 \cdot 10^{-8}$ (for the remaining expressions with densities and for sums $^{1}S_H$, $^{2}S_H$, and $^{\nu}S_H$ this requirement is more convenient). Thus we have it that the requirement for precision of 0.01" up to a zenith distance of 76° for the refraction $\Delta R_{\Delta H}$ and ΔR_H cannot always be carried out.

3.2 Evaluation of Some Formulas for the Refraction ΔR_H and $\Delta R_{\Delta H}$

Formula (3.3) and formula (3.1) as well, were first compared with the help of the numerical values with a similar formula adduced in [1, p. 19]. Oterma introduces here for ΔR (here designated ζ) the potential series sec z', and he replaces the standard atmosphere of the earth by a simple course of the temperature and the pressure with altitude. The values and their differences were calculated for the separate strata and for the zenith difference z' = 60, and they are given in Table 3.1. The expressions for H = 100 km should already by practically identical with R_∞, i.e., with the values in Table 2.2 in the next to the last and the last paragraphs for z' = 60°. For the formulas derived here the difference is practically zero and confirms the correctness of the equation (3.3). For the formula in [1] it amounts to 0.33". The differences [1] - (3.3) in the Table 3.1 show systematic variations and could be effected through different numerical values of the introduced constants or with the adjusting of some formula.

TABLE 3.1.

H	[1]	Eq. (3.3)	Difference [1] (3.3)
km			
10	65.269″	65.170″	0.099″
20	91.287	91.289	0.002
30	96.943	96.993	0.050
40	98.044	98.060	0.016
50	98.272	98.296	0.024
60	98.330	98.398	0.068
80	98.354	98.424	0.070
100	98.356	98.425	0.069
150	98.356	98.425	0.069

TABLE 3.2.

H	Baldini [2]	Eq. (3.3)	Difference [2] − (3.3)
km			
10	64,983″	65,170″	−0,187″
20	87,046	91,289	−4,243
30	94,536	96,993	−2,457
40	97,065	98,060	− 0,995
50	97,920	98,296	−0,376
60	98,215	98,398	−0,183
80	98,342	98,424	−0,082
100	98,362	98,425	−0,063
150	98,362	98,425	−0,063

Let us compare the formula (3.3) and by means of the same also the formula (3.1) further with a similar formula in the publication [2]. Baldini uses here the potential series tg z' and finds the relation for dn:n mit the help of the Gladston-Dale law and the empirical relationship between density and elevation

$$\varrho = \varrho_0 \, e \exp(-0{,}1082H), \qquad (3.5)$$

where e is the basis of the natural logarithm. As the height over which no refraction of the light ray occurs he considers 64-km height up to a zenith distance of 75°. He gets more or less a composite formula [2], where he defines the refraction ΔR_H up to a definite height H simply as a function of the height H. The Table 3.2 presents the comparison of this formula with the formula (3.3) once more for z' = 60°. The details resulting from the comparison in

the Table 3.2 are for the higher strata the same as those from the comparison done in Table 3.1. For heights up to 60 km, however, the differences are rather great. This disagreement is caused through the relationship (3.5), which does not comprehend the conditions of the earth's standard atmosphere, and even varies from some extreme (Part 4) applied models of the earth's atmosphere [12].

On the whole it turns out also that the determination of the refraction ΔR_H and $\Delta R_{\Delta H}$ is more difficult than for the refraction R_∞ and that it is more to the point from here on in ΔR_H and $\Delta R_{\Delta H}$, and through this also in the parallactic refraction, to require a precision of 0.1".

4. Interpretation of the Refraction R_∞, $\Delta R_{\Delta H}$, and ΔR_H on Various Models of the Earth's Atmosphere

4.1 Influence of Time of the Year and the Latitude

1. In [4, p. 56] the data for the density every 5 km are given and at that in a stratum of 25 km up to 90 km for the (a) low latitude $\phi = 10°$, (b) the high latitude $\phi = 58°$ in the summer, and (c) the high latitude $\phi = 58°$ in winter.

TABLE 4.11.

z'	$\varphi = 10°$	$\varphi = 58°$		Standard atmosphere	
		Summer	Winter	exact	approximate
45°	1,76"	1,85"	1,48"	1,89"	1,88"
70°	4,82	5,08	4,06	5,20	5,15

The data of the normal refraction (for $t_0 = 15°C$ and $p_0 = 760$ mm Hg) $\Delta R_{\Delta H}$ were calculated in accordance with (3.1) and compared with the refraction determined from [4] for the standard atmosphere for $z' = 45°$ and 70° (Table 4.11). The refraction is here more influenced by the time of the year than by the different latitudes. Thus it has higher values for the summer than for the winter. The model at (b) was the nearest to the standard atmosphere. For it a two-fold computation was carried out, a precise one with

close gradation for each 1 km [11] and an approximate one with a gradation of every 5 km in elevation (see Table 2.1). There are no great differences.

TABLE 4.12.

z'	ΔR_H: 0 to 65.5 km		$\Delta R_{\Delta H}$: 18,05 to 89,96 km	
	$\varphi = 33°$ S	Stand. Atm.	$\varphi = 65,6°$ N	Stand. Atm.
45°	56,94"	56,96"	5,14"	5,58"
70°	155,26	155,30	13,81	14,99
Date	18. 11. 1957		16. 2. 1961	

2. In a similar way the normal refraction ΔR_H in the layer from 0 to 65.5 km for 33° southern latitude and the normal refraction $\Delta R_{\Delta H}$ in the layer from 18.05 to 89.96 km northern latitude [4, p. 49, 50, and 54] were computed and compared with the corresponding refractions for the standard atmosphere (Table 4.12). It is hard to judge whether a high latitude has an effect on the $\Delta R_{\Delta H}$ (ϕ = 65.6°N). Rather it would be the influence of the winter period, since the difference from the standard atmosphere had the same sign as in the Table 4.11. The situation is similar for ΔR_H (ϕ = 33°S) where there was agreement with the Table 4.11 for the summer.

4.2 Influence of the Maximum and Minimum Yearly Temperature on ΔR_H and R_∞

According to information in [13] for the year 1960 the refraction values for the maximum t_{max} = 26.6°C and for the minimum t_{min} = -17.4°C of the yearly temperature and for the average pressures p in the year 1960 in the stratum from 0 to an altitude of 20 km were totaled with the aid of the formula (3.3). The values $\Delta R_{H/t,p}$ calculated for the temperature t_{max} and t_{min} and for the normal temperature t_0 and the pressure p_0 in the location of the station and in fact for both cases. The differences were relatively small and of opposite sign, as is the case in Table 4.11. The value for the standard atmosphere is in between.

TABLE 4.2.

z'	t_{max}	t_{min}	Stand. Atm.
45	52,63"	53,01"	52,85"
70	143,74	143,65	144,31

With the bringing together of the information for the maximum temperature (Table 4.2) with the information for the summer (Table 4.11), and similarly for the minimum temperature and for the winter and adding on influence of the standard atmosphere above the altitude of 90 km we get the values for the astronomical refraction R_∞, which do not differ from each other.

4.3 Influence of the Average Monthly Temperatures and Pressures on ΔR_H

Similarly as in 4.2 there were obtained here in accordance with [13] for the year 1960, the values of the normal refraction ΔR_H for the average monthly temperatures and pressures, and in fact from 0 to H = 15 km. In the Table 4.3 the differences ΔR_H are much more expressive than in the Tables 4.2, 4.11, and 4.12. This is brought about apparently by the greater oscillations in the air density in the upper stratum H = 15 km, which to a great extent are expressed by the separation of the densities in the stratum. This comes about through the fact that the heat and cold relationships in the upper layers are more stable than in the lower strata. The values fit well with the refraction value for the standard atmosphere, except the anomalous values in April and September. In the graphic representation there is generally a sinus curve expressed.

TABLE 4.3. $\phi = 50°$, 0^h CET (Central European Time), H = 15 km.

Month \ z'	45°	70°	Month \ z'	45	70°	Month \ z'	45°	70°
I	48,33"	132,04"	V	47,95"	131,02"	IX	46,13"	126,08"
II	48,30	131,95	VI	47,56	129,97	X	47,95	131,03
III	48,28	131,91	VII	47,84	130,74	XI	48,02	131,23
IV	46,84	128,00	VIII	47,71	130,40	XII	48,87	133,52
			Standard atmosphere	47,94	131,02			

In the astronomical refraction value R_∞ the influence of the average monthly temperatures and pressures does not find expression. However, it finds expression with its full value in measurements to high targets (up to an elevation of about 25 km).

4.4 Application of the Formulas for ΔR_H and for R_∞ to the Instantaneous Status of the Atmosphere

In accordance with [14, 15] with the help of temperature gradients the temperatures at the station in western Tatra (1300 m above sea level) were lined up with the temperature values obtained in Poprad. The temperature interpolation was carried out linearly and at that in such a way that for the layer above the station from 200 m up to the elevation H $\doteq$ 20 km the temperatures from the ascent measurement were already left as they were. The application was undertaken for triple measurements (Table 4.41). Variations of the refraction ΔR_H from the refraction gotten from the standard atmosphere here also have the same sign as for all cases of the summer measurements in the preceding Tables 4.11 to 4.3. If we complete the remaining layer with the standard atmosphere we establish the astronomical refraction R_∞ (Table 4.42).

TABLE 4.41.

	25. 8. 1960	H = 21,1 km	3. 9. 1961	H = 18,1 km	3. 9. 1961	H = 18,1 km
z	12^h CET	Stand. Atm.	12^h CET	Stand. Atm.	18^h CET	Stand. Atm.
45°	53,34"	53,53"	51,20"	51,45"	51,07"	51,39"
70°	145,61	146,11	139,81	140,50	139,44	140,36

TABLE 4.42.

z	25. 8. 1960 12^h CET	3. 9. 1961 12^h CET	3. 9. 1961 18^h CET	Stand. Atmosphere
45°	56,96$_5$"	56,96$_6$"	56,96$_5$"	56,96$_5$"
70°	155,32	155,33	155,32	155,32

With the completion of the atmosphere the certain anomalous density ρ at the elevation H (ascending) thus did not make itself

count; it went namely into the sum ${}^{1}S$, ${}^{2}S$, and ${}^{\nu}S$ where it comes to be expressed to a far lesser extent, and instead of it really the value $\rho = 0$ comes up which is absolutely correct.

4.5 Influence of Different Meteorological Situations and the Direction of the Light Ray on the Refraction R_∞ and ΔR_H

There were selected two different meteorological situations for the Central-European area: (a) an unstable and complicated meteorological situation (30 January 1960), and (b) a stable meteorological situation (5 May 1960). For these two situations for 0^h CET instantaneous atmosphere models up to 13 km were compiled with the help of ascension measurements (in Prague, Poprad, Dresden, Berlin, and Munich [13]) and the elevations of levels of equal air density were determined. Because these do not agree, the influence of the locating azimuth on the light beam was checked. Two light beams in $z' = 70°$ were selected, and in fact the first in the direction north and the second in the direction south. While the variations in the heights of the level of like density in the path of the light ray in the case (a) go as far as 60 m, for (b) this difference is less. They show up in the total ${}^{1}S$ with the value of $2 \cdot 10^{-9}$, something that does not influence the total refraction R_∞. So also for ΔR_H and for the above given case the dependence at azimuth is equal to zero if indeed the locating of reference points is done approximately at the same time.

Evaluation of Part 4: It is proved that the different earth-atmosphere models characterized by different latitude, time of the year, synoptical situation, or momentary position do not influence the astronomic refraction R_∞ with any real measurable values. It is brought about by the fact that the density ρ in equation (3.3) attains zero value, so that anomalous densities only appear in the totals (3.4), where they do not show up to the same extent as when the body is inside of the atmosphere, with the exception that the expressions (3.4) do not take any value at all. The establishing of the fact that R_∞ does not depend on the instantaneous state of the atmosphere is in line with the circumstance also that a refraction angle mainly does not depend on the atmospheric situation of the station

and the target. The target, however, is in infinity and the density at the station is determined with a relatively significant precision. It is further established that the refraction $\Delta R_{\Delta H}$ and the refraction ΔR_H, which enter into the parallactic reaction with their full value, in contrast to the instantaneous state of the atmosphere, i.e., the distribution of the temperature and the pressures in the atmosphere in the different yearly and monthly periods, are dependent on the meteorological situation and thus on the time. This dependence causes for $z' = 45°$ in refraction under 50 km about 0.1" to 2" and for $z' = 70°$ already around 1" to 7". For higher temperatures in the atmosphere it was as a rule somewhat smaller than the refraction derived from the standard atmosphere. So what holds good here is that the lower the target (body M) is, the greater the influence of the momentaneous distribution of the densities on the reaction will be. This amounts mainly to the fact that ΔR_H depends on the density ρ at the height of the target, and resulting from this that the temperature and pressure relationships are less evened out in the lower strata than in the higher ones. On the other hand a dependence on the latitude and the azimuth was not shown. The small number of tested cases should be designated as inadequate.

The system of computation by way of numerical integration or with the help of tabulation of the sums ${}^1S_H$, ${}^2S_H$, and ${}^{\nu}S_H$ [11] is clear and rapid. One also thereby gets rid of the less common assumption of a spherical form of an air layer (page 1). The gradation of the argument of the density ρ can be selected sufficiently fine.

Another unsolved problem here is th[illegible]luence of the lower strata of the atmosphere on the refraction, as it is dealt with, for example in [16].

5. Parallactic Refraction $\bar{R}_H$

5.1 Basic Relationship for $\bar{R}_H$

The parallactic refraction $\bar{R}_H$ for the station S and the body M at the elevation H we will define with the help of the designation

$$\bar{R}_H = R_\infty - \Delta R_H + \xi, \qquad (5.1)$$

where ξ is the parallactic angle at the apex M (Fig. 2). $\bar{R}_H$ is thus the angle that the chord C (connecting line SM) and the direction b to the star A enclose. In other words it is the difference between the real zenith distance of the body M and the star A, the images of which are identical to one from the station S. The parallactic refraction is always less than the astronomical. It becomes less with the increase in the elevation H. In what follows for higher targets [17] we will determine meteors and satellites. The difference $R_\infty - \Delta R_H$ is really equal to the refraction $\Delta R_{\Delta H}$ of the light ray in passing through the air layer, the lower boundary of which is the height of the body M. That means it is $H_u = H$ and the density $\rho_u = \rho$ while the upper boundary above all the boundaries increases (practically the computations were carried out up to an elevation of 200 km) and the density $\rho_o = 0$. For this reaction case we introduce a new corresponding symbol ΔR^H. In Fig. 2 it is the angle between

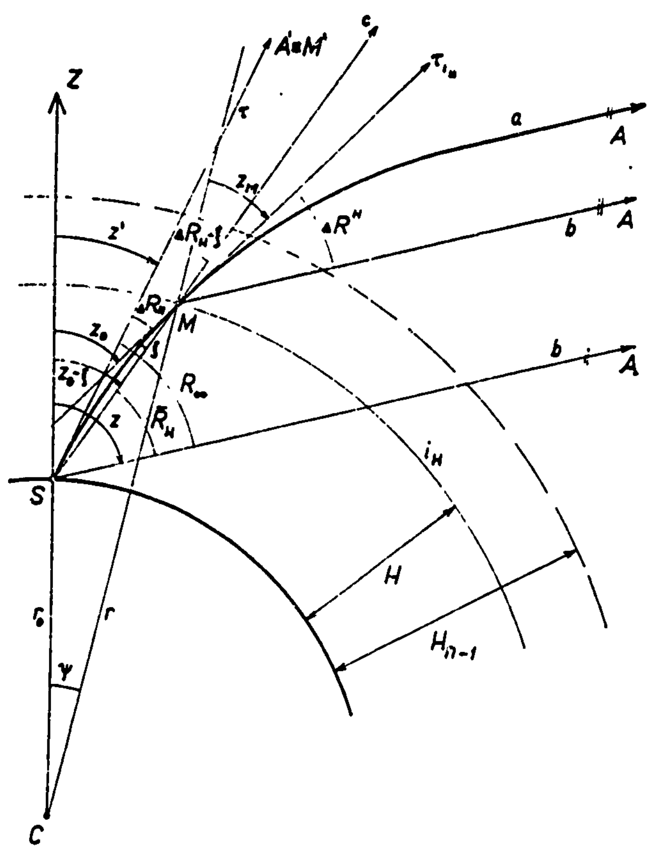

Fig. 2.

the tangent τ_{1_H} and the direction b to the star A. Then it is $\Delta R^H = R_\infty - \Delta R_H$ and the formula (5.1) becomes the form $\overline{R}_H = \Delta R^H + \xi$. From [3] one has

$$\operatorname{tg} \xi = r_0 (nr)^{-1} q \operatorname{cosec} \psi \sin z_0 [1 + r_0 (nr)^{-1} q \operatorname{cosec} \psi \cos z_0]^{-1}. \qquad (5.2)$$

In accordance with the formula (5.2) we can calculate ξ for as small an amount as we wish. The required precision is also from here on 0.1". For the body M in higher strata, i.e., from 50 km on up, the expression (5.2) is to be replaced by the form

$$\xi = r_0 (nr)^{-1} q \sin z_0 \operatorname{cosec} \psi \qquad (5.3)$$

The expression $q = n_0 \sin z' - n \sin z_0$ is adjustable. We introduce, however, $n_0 : n = 1 + \nu$ and $z_0 = z' + R_\infty - \Delta R^H = z' + \Delta R_H$ (Fig. 2). Then there is

$$\begin{aligned} q = n \sin z' [\nu + 1/2!\, \Delta R_H^2 - 1/4!\, \Delta R_H^4 + 1/6!\, \Delta R_H^6 - \\ - \operatorname{cotg} z' (\Delta R_H - 1/3!\, \Delta R_H^3 + 1/5!\, \Delta R_H^5 - 1/7!\, \Delta R_H^7)] \end{aligned} \qquad (5.4)$$

where ΔR_H is given with the equation (3.3). The formula (5.4) corresponds with its precision of 0.1" for as little elevation as one may wish, for z' from 0° to 76° and for cosec ψ up to 10^4. The angle ψ can be determined from the expression $\psi = z_0 - z_M$ where we get z_M from the expression $\sin z_M = n_0 r_0 (nr)^{-1} \sin z'$ and n from the law $n = 1 + k\rho$. The expression ψ can also be gotten from the spherical triangle with the apices: zenith Z of the station S, body M, and the earth's pole P from the proposition $\cos \psi = \sin \phi_0 \sin \delta_0 + \cos \phi_0 \cos \delta_0 \cos t_0$, where ϕ_0 is the geocentric latitude of the station S, δ_0 the geocentric declination of the body M, and t_0 the geocentric hour angle of the body M.

For the case where the elevation of the body M is greater than about 70 km one has $\rho = 0$, $n = 1$, $\Delta R^H = 0$, and hence $\overline{R}_H = \xi$. Then one has $z_0 = z' + R_\infty$, and for the parallactic refraction there is valid the relationship $\overline{R}_H = 483.95'' \, r^{-1}_{[km]} \sin^2 z' \operatorname{cosec} \psi \, [1 + 0.9961 \cdot \operatorname{tg}^2 z' - 305 \cdot 10^{-5} \operatorname{tg}^4 z' + 14 \cdot 10^{-6} \operatorname{tg}^6 z']^5$.

In [1] on the one hand the calculation was through continuous approximation and on the other hand through series. The bodies were weighed within the air and outside of it. On comparison with the formulas in [1], for example, for H = 20 km and for z' = 60° the differences were not more than 0.1". The advantages of these formulas is the greater simplicity of the form than in [1]. The disadvantage is the required high precision of the values entering into the computation.

6. Tabulation of Some Values

For illustration and practical application in this part some values in the dependence on the elevation H and the zenith distance z' are tabulated. Values in the lower elevations do not adapt themselves to tabulation because of the very considerable variations.

6.1 Influences of Light-Ray Diffraction

6.11 Correction of the Elevation above Sea Level of the Body M from the Influences of Light-Ray Diffraction in Pass-Through Masses of Air

In Fig. 3 M represents the correct position of the body and M' the one doubtful because of the bending of the light ray. The correct elevation above sea level is $H = H' + o_H$, where H' is the zenith distance, z' the corresponding doubtful elevation, and o_H the correction of the elevation H' and which is always negative. For the derivation of this correction we proceed from the sinus proposition $(r_0 + H) \sin (z' + \Delta R_H - \xi - \psi) = r_0 \sin (z' + \Delta R_H - \xi)$ and $(r_0 + H') \sin (z' - \psi) = r_0 \sin z'$. Through the proper adjustment and subtraction of these equations we get the respective correction $o_H = -30.9 \cdot (\Delta R_H - \xi)'' \sin \psi \sin^{-2} (z' - \psi)$, where for r_0 the value 6368.8 km was taken. The value z' is either known from measured values or can be known from the nautical triangle M', Z, P (the center of the unit sphere is the station S). In Table 6.1 the corrections o_H are given for various z' and H'. Let us note again that it is a matter of the influence of the angle $\Delta R_H - \xi$ (Fig. 3), i.e., the influence of the error in the measured zenith distance.

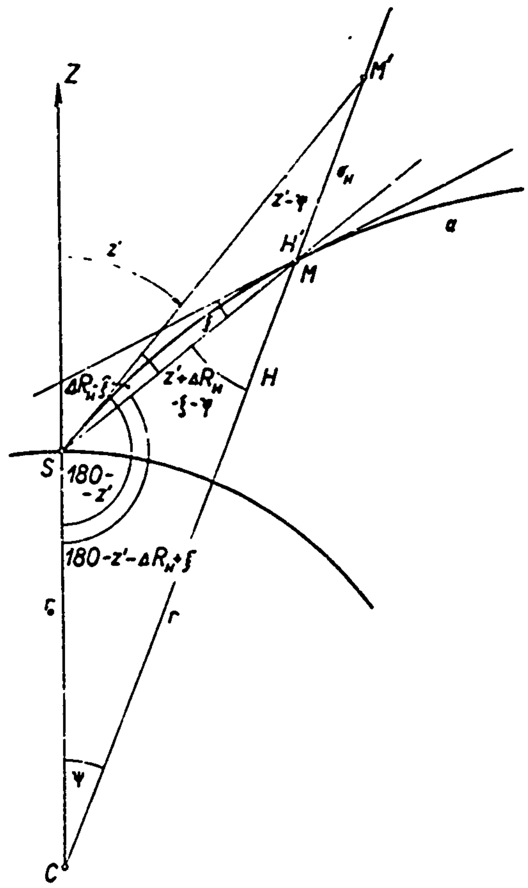

Fig. 3.

TABLE 6.1.

z' / H'	10	20	30	40	50	60
km	m	m	m	m	m	m
10	0	0	0	0	−1	−1
20	−3	−3	−4	−5	−9	−14
30	−6	−7	−8	−10	−14	−23
40	−7	−10	−11	−14	−20	−34
50	−12	−13	−15	−19	27	−45
60	−14	−16	−18	−23	34	−56
80	−20	−22	−26	−33	−48	−78
100	−26	−29	−34	−43	−61	−100

TABLE 6.3.

H \ z	5	10	20	40	60
150 km	0,3"	0,4"	1,0"	2,6"	5,8"
160	0,2	0,4	0,9	2,5	5,4
180	0,2	0,3	0,8	2,2	4,8
200	0,2	0,3	0,7	2,0	4,4
220	0,2	0,3	0,7	1,8	4,0
240	0,2	0,2	0,6	1,7	3,7
260	0,2	0,2	0,6	1,5	3,4
280	0,2	0,2	0,5	1,4	3,2
300	0,1	0,2	0,5	1,3	3,0
320	0,1	0,2	0,5	1,2	2,8
340	0,1	0,2	0,4	1,2	2,6
360	0,1	0,2	0,4	1,1	2,5
380	0,1	0,2	0,4	1,0	2,4
400	0,1	0,2	0,4	1,0	2,3
450	0,1	0,1	0,3	0,9	2,0
500	0,1	0,1	0,3	0,8	1,8

6.12 Influence of the Diffraction of the Light Beam on Passing Through the Air Space onto the Position of the Body

In the following by means of tables we give the values: ΔR_H – refraction skywards H, ξ – parallactic angle, $\bar{R}_H$ – parallactic refraction. The Talbe 6.2 serves for obtaining concrete presentations of angle sizes ΔR_H, ξ, and $\bar{R}_H$.

6.2 Computation of the Parallactic Refraction for Some Satellite Types

In accordance with the formula (5.3) there were computed the values $\bar{R}_H = \xi$ for some elevations in the interval from 150 km to 500 km and for different zenith distances (Table 6.3). Within these elevation distances ordinarily there move the perigee and the apogee of the Soviet satellites. All the values in the Part 6 are calculated for the normal temperature t_0 = 15°C and for the normal pressure p_0 = 760 mm Hg.

TABLE 6.2.

H \ z		10	20	30	40	50	60°
20 km	ΔR_H	9,3"	19,2"	30,5"	44,3"	62,9"	91,3"
	ξ	4,4	8,3	13,1	18,9	26,6	38,4
	$\bar{R}_H$	5,1	9,8	15.5	22,3	31,6	45,5
40 km	ΔR_H	10,0"	20,7"	32,8"	47,6"	67,6"	98,1"
	ξ	3,2	5,0	7,8	11,2	15,7	22,9
	$\bar{R}_H$	3,3	5,1	7,9	11,4	16,0	23,0
60 km	ΔR_H	10,1"	20,7"	32,9"	47,8"	67,8"	98,4"
	ξ	1,6	3,7	5,8	8,4	10,8	14,3
	$\bar{R}_H$	1,6	3,7	5,8	8,5	10,9	14,3
80 km	ΔR_H	10,1"	20,8"	32,9"	47,8"	67,8"	98,4"
	ξ	1,2	2,4	3,8	5,7	7,5	10,7
	$\bar{R}_H$	1,2	2,4	3,8	5,7	7,6	10,7
100 km	ΔR_H	10,1"	20,8"	32.9"	47,8"	67,8"	98,4"
	ξ	0,9	1,9	3.1	4,6	6,1	8,6
	$\bar{R}_H$	0,9	1,9	3,1	4,6	6,1	8,6
120 km	ΔR_H	10,1"	20,8"	32,9"	47,8"	67,8"	
	ξ	0,8	1,7	2,7	3,7	5,1	
	$\bar{R}_H$	0,8	1,7	2,7	3,7	5,1	
140 km	ΔR_H	10,1"	20,8"	32,9"	47,8"	67,8"	
	ξ	0,8	1,5	2,3	3,2	4,4	
	$\bar{R}_H$	0,8	1,5	2,3	3,2	4,4	

7. Prologue

Briefly one can affirm that the astronomical refraction R_∞ is not dependent on the instantaneous state of the atmosphere if we naturally take into consideration the state of the atmosphere in the lower strata. The expressions $\Delta R_{\Delta H}$ and ΔR_H, however, depend on the instantaneous state of the atmosphere, and hence on the time. Deviations from the standard atmosphere can cause in these expressions series-wise deflections of 1" for the zenith distance of 45° (Part 4). The influence depends in indirect proportion on the height of the object. It is, however, dependent in direct proportion on the density

of the air, or on its changes, which are greater in the lower strata than in the higher ones. The same holds true for the expression ΔR^H occurring in the formula for the parallactic refraction (Part 5). One can lend to this expression various forms depending on what target elevation is involved. It could conceivably be possible to omit it altogether if the target is outside of the atmosphere, even if in infinity. The parallactic refraction then is directly equal to the parallactic angle ξ, a situation which already holds good for elevations above 70 km. Finally one can affirm that it is necessary to introduce their values as corrections for refining the positions of the satellites, meteors, and high targets, as is shown by Tables 6.2 and 6.3. Besides, there occur here the influences of the instantaneous state of the air which are described in Part 4.

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Footnotes

[1]Address: Karlovo nám. 13, Praha 2 - Nové Město.

[2]$R_\infty = R_0 + \Delta R_t(t - t_0) + \Delta R_b(p - p_0)$, where $t_0 = +15.0°C$ and $p_0 = 760$ mm Hg.

[3]For the case where the body is inside the atmosphere we have $\Delta R_H = R_\infty$.

[4]For the conditions at the station we consider further only a normal temperature t_0 and a normal pressure p_0, as well as the normal density ρ_0.

[5]The change in the value $\bar{R}_H$ with the temperature and pressure is described in [3].